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MAŠINSKI FAKULTET U KRAGUJEVCU
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Kragujevac, Srbija

31. SAVETOVANJE PROIZVODNOG MAŠINSTVA SRBIJE I CRNE GORE SA MEĐUNARODNIM UČEŠĆEM

31th CONFERENCE ON PRODUCTION ENGINEERING OF SERBIA AND MONTENEGRO WITH FOREIGN PARTICIPANTS

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Kragujevac, 19. - 21. septembar 2006.

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PREDGOVOR

Prvo Savetovanje proizvodnog mašinstva Jugoslavije održano je u Beogradu 1965. na inicijativu prof. dr Vladimira Šolaje, kada je i formirana Zajednica naučno-istraživačkih institucija proizvodnog mašinstva, koju su sačinjavali mašinski fakulteti i istraživački instituti iz skoro svih republičkih centara tadašnje države. Zajednicu proizvodnog mašinstva SCG, u vreme pripreme Savetovanja, sačinjavaju: Mašinski fakultet u Beogradu, Mašinski fakultet u Nišu, Mašinski fakultet u Kragujevcu, Mašinski fkaultet u Podgorici, Institut za proizvodno mašinstvo FTN iz Novog Sada, Institut za industrijske sisteme FTN iz Novog Sada, Tehnički fakultet u Čačku, Mašinski fakulet u Kraljevu, LOLA Institut u Beogradu i Mašinski fakultet u Prištini.

31. Savetovanje proizvodnog mašinstva SCG održava se u Kragujevcu, u organizaciji Katedre za proizvodno mašinstvo Mašinskog fakultet u Kragujevcu. Prethodna Savetovanja u Kragujevcu su održana 1969. (5. Savetovanje) i 1985. godine (19. po redu).

I ovo Savetovanje, kao i nekoliko prethodnih, održava se u vreme intenzivnih društvenih promena, značajnih za šire aspekte proizvodnog mašinstva. Vlasnička transformacija i oživljavanje privrede u proizvodnim oblastima, posebno u metaloprerađivačkoj industriji, na samom je početku. Privatizacija i pokretanje proizvodnje u velikim industrijskim sistemima sprovodi se sporo i necelovito. Prema društvenim planovima, završetak transformacije u ovoj oblasti se najavljuje za kraj 2007., kada bi trebalo očekivati i značajnije pokretanje prizvodnih delatnosti.

Na ovom Savetovanje, organizovanom za samo godinu dana, biće izloženo 120 radova autora iz Srbije i Crne Gore i inostranstva (Ukrajina, Slovačka, Poljska, SAD, Slovenija, Bosna i Hercegovina, Hrvatska, Makedonija). Aktivnosti na Savetovanju će se obavljati u više sekcija, koje obuhvataju sledeće tematske oblasti: Proizvodne tehnologije, obradne sistemi i materijale; Upravljanje proizvodnim sistemima, razvoj proizvoda i CAx tehnologije; Tribologiju, revitalizaciju, reinžinjering i održavanje; Menadžment kvalitetom i ekološke tehnologije.

Pored osnovnog zadatka Savetovanja - upoznavanje se trenutnim stanjem istraživanja u oblasti proizvodnog mašinstva, nadamo se da će saopšteni rezultati i diskusija na okruglom stolu, doprineti u definisanju strategije razvoja ove, izuzetno značajne oblasti za dalji privredni razvoj naše države.

Zahvaljujemo se svim domaćim i stranim autorima, članovima recenzetskog tima, kao i institucijama i pojedincima, koji su doprineli kvalitetnoj relizaciji programa Savetovanja.

Kragujevac,
19. 09. 2006.

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SADRŽAJ

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B1 RAZVOJ PROIZVODA
PRODUCTS DEVELOPMENT

B2 CAx TEHNOLOGIJE
CAx TECHNOLOGIES

[illegible]

C2: REVITALIZACIJA, REINŽINJERING, ODRŽAVANJE
REVITALISATION, REENGINEERING AND MAINTAINING

[illegible]

D2: EKOLOŠKE TEHNOLOGIJE
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THE CONTACT PROBLEMS BASED ON THE PENALTY METHOD

S. Vulović [1], M. Živković [2], N. Grujović [3], V. Mandić [4]

Abstract: *In this paper the contact problems including frictional effects arepresented. The friction forces are assumed to follow the Coulomb law, with a slip criterion treated in the context of a standard return mapping algorithm. The algorithm is amenable to exact linearization and asymptotic quadratic rate of convergence can be achieved within a Newton-Raphson iterative solution scheme.*

Solution results for verification example are presented at the end of this paper.
Key words: *contact problem, penalty method, friction*

1. INTRODUCTION

Numerical analysis of frictional contact problems has been one of the research topics of main interest in recent years. Frictional contact problems arise in many application fields such as metal forming processes, the impact of cars, etc.

The effective application of finite element contact solvers need a high degree of experience since the general robustness and stability cannot be guaranteed. For this reason the development of more efficient, fast and stable finite element contact discretizations is still a hot topic, especially due to the fact that engineering applications become more and more complex.

In this paper, framework for contact problems with friction is developed based on the penalty method. The penalty formulation has the advantage that it is purely geometrically based and therefore no additional degrees of freedom have to be activated or inactivated. Numerical example is shown to demonstrate that the presented algorithm can be successfully applied to contact problems.

2. CONTACT KINEMATICS

As the configurations of two bodies coming into the contact are not a priori known, contact represents a nonlinear problem even when the continuum behaves as a linear elastic material.

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Using a standard notation in contact mechanics, for each pair of contact surfaces, involved in the problem, we will define slave ($\Gamma_C^{(1)}$) and master surfaces ($\Gamma_C^{(2)}$), Fig. 1. The condition which must be satisfied is that any slave particle cannot penetrate the master surface.

Let $\bar{\mathbf{x}}$ be the projection point of the current position of the slave node $\mathbf{x}^k$ onto current position of the master surface $\Gamma_C^{(2)}$, defined as

$$\frac{\mathbf{x}^k - \bar{\mathbf{x}}(\bar{\xi}^1,\bar{\xi}^2)}{\left\|\mathbf{x}^k - \bar{\mathbf{x}}(\bar{\xi}^1,\bar{\xi}^2)\right\|}\cdot \bar{\mathbf{a}}_\alpha(\bar{\xi}^1,\bar{\xi}^2) = 0 \tag{1}$$

where $\alpha = 1,2$ and $\bar{\mathbf{a}}_\alpha(\bar{\xi}^1,\bar{\xi}^2)$ are the tangent covariant base vectors at the point $\bar{\mathbf{x}}$. The normal gap or the penetration g_N for slave node k is defined as the distance between current positions of this node to the master surface $\Gamma_C^{(2)}$:

$$g_N = (\mathbf{x}^k - \bar{\mathbf{x}})\cdot \bar{\mathbf{n}} \tag{2}$$

where $\bar{\mathbf{n}}$ refers to the normal to the master face $\Gamma_C^{(2)}$ at point $\bar{\mathbf{x}}$ (Fig. 1). Normal will be defined using tangent vectors at the point $\bar{\mathbf{x}}$

$$\bar{\mathbf{n}} = \frac{\bar{\mathbf{a}}_1 \times \bar{\mathbf{a}}_2}{\left\|\bar{\mathbf{a}}_1 \times \bar{\mathbf{a}}_2\right\|} \tag{3}$$

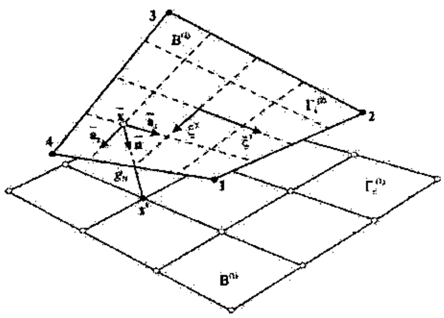

Figure 1. Geometry of the 3D node-to-segment contact element

This gap (2) gives the non-penetration conditions as follows

$g_N = 0$ perfect contact; $g_N > 0$ no contact; $g_N < 0$ penetration (4)

If the analyzed problem is frictionless, function (4) completely defines the contact kinematics. However, if friction is modeled, tangential relative displacement must be introduced. In that case, the sliding path of the node $\mathbf{x}^k$ over the contact surface $\Gamma_C^{(2)}$ is described by total tangential relative displacement as

$$g_T = \int_{t_0}^{t}\left\|\dot{\mathbf{g}}_T\right\| dt = \int_{t_0}^{t}\left\|\dot{\bar{\xi}}^\alpha \bar{\mathbf{a}}_\alpha\right\| dt = \int_{t_0}^{t}\sqrt{\dot{\bar{\xi}}^\alpha \dot{\bar{\xi}}^\beta a_{\alpha\beta}}\, dt \tag{5}$$

in time interval from t_0 to t.

The time derivatives of parameter $\bar{\xi}^\alpha$ in equation (5) can be computed from (1), [7]. We obtain the following result

$$\bar{a}_{\beta\alpha}\dot{\bar{\xi}}^{\beta} = \left[\dot{\mathbf{x}}^{k} - \dot{\bar{\mathbf{x}}}\right]\cdot\bar{\mathbf{a}}_{\alpha} = \dot{g}_{T\alpha} \tag{6}$$

where $\bar{a}_{\alpha\beta} = \bar{\mathbf{a}}_{\alpha}\cdot\bar{\mathbf{a}}_{\beta}$ is the metric tensor in point $\bar{\mathbf{x}}$ of the master surface $\Gamma_C^{(2)}$. From the equations (5) and (6) we can express the relative tangential velocity at the contact point

$$\dot{\mathbf{g}}_T = \dot{\bar{\xi}}^{\alpha}\bar{\mathbf{a}}_{\alpha} = \dot{g}_{T\alpha}\bar{\mathbf{a}}^{\alpha} \tag{7}$$

3. CONSTITUTIVE EQUATIONS FOR CONTACT INTERFACE

A contact stress vector $\bar{\mathbf{t}}$ with respect to the current contact interface $\Gamma_C^{(2)}$ can be split into a normal and tangential part.

$$\bar{\mathbf{t}} = \bar{\mathbf{t}}_N + \bar{\mathbf{t}}_T = \bar{t}_N\bar{\mathbf{n}} + \bar{t}_{T\alpha}\bar{\mathbf{a}}^{\alpha} \tag{8}$$

where $\bar{\mathbf{a}}^{\alpha}$ is contravariant base vector. The stress acts on both surfaces according to the action-reaction principle: $\bar{\mathbf{t}}(\bar{\xi}^1,\bar{\xi}^2) = -\mathbf{t}$ in the contact point $\bar{\mathbf{x}}$. The tangential stress $\bar{t}_{T\alpha}$ is zero in the case of frictionless contact. In the case of contact there is condition $\bar{t}_N < 0$. If there is not penetration between the bodies, then relations $g_N > 0$ and $\bar{t}_N = 0$ hold. This leads to the statements

$$g_N \geq 0, \quad \bar{t}_N \leq 0, \quad \bar{t}_N g_N = 0 \tag{9}$$

which are known as Kuhn-Tucker conditions. Using the penalty method for normal stress, constitutive equation can be formulated as

$$t_N = \varepsilon_N g_N \tag{10}$$

where ε_N is the normal penalty parameter.

In tangential direction there is difference between stick and slip. As long as no sliding between two bodies occurs, the tangential relative velocity is zero. If the velocity is zero, also the tangential relative displacement (5) is zero. This state is called stick case with the following restriction:

$$\dot{\mathbf{g}}_T = \mathbf{0} \Leftrightarrow \mathbf{g}_T = \mathbf{0} \tag{11}$$

For stick a simple linear constitutive model can be used to describe the tangential stress.

$$t_{T\alpha}^{stick} = \varepsilon_T g_{T\alpha} \tag{12}$$

where ε_T is the tangential penalty parameter.

A relative movement between two bodies occurs if the static friction resistance is overcome and the loading is large enough such that the sliding process can be kept. The tangential stress vector is restricted as follows:

$$t_{T\alpha}^{sl} = -\mu\left|\mathbf{t}_N\right|\frac{\dot{g}_{T\alpha}^{sl}}{\left\|\dot{\mathbf{g}}_T^{sl}\right\|} \tag{13}$$

where μ is friction coefficient. In the simplest form of Coulomb's law (13), μ is constant so there is no difference between static and sliding friction.

After the introduction of the stick and slip constraints, we need to introduce indicator to define whether stick or slip actually take place. Therefore an indicator function

$$f = \left\|\mathbf{t}_T\right\| - \mu\left|t_N\right| \tag{14}$$

is evaluated, which respect the Coulomb's model for frictional interface law. In the equation (14) the first term is $\|\mathbf{t}_T\| = \sqrt{t_{T\alpha}\bar{a}^{\alpha\beta}t_{T\beta}}$.

A backward Euler integration scheme and return mapping strategy are employed to integrate the friction equations (14). If a state of stick is assumed, the trial values of the tangential contact pressure vector $t_{T\alpha}$, and the indicator function f at load step $n+1$ can be expressed in terms of their values at load step n as follows

$$t^{trial}_{T\alpha\ n+1} = t_{T\alpha\ n} + \varepsilon_T \Delta g_{T\alpha\ n+1} = t_{T\alpha\ n} + \varepsilon_T \bar{a}_{\alpha\beta} \Delta\xi^{\beta}_{n+1} \tag{15}$$

$$f^{trial}_{Tn+1} = \left\|\mathbf{t}^{trial}_{Tn+1}\right\| - \mu\left|t_{Nn+1}\right| \tag{16}$$

The return mapping is completed by

$$t_{T\alpha\ n+1} = \begin{cases} t^{trial}_{T\alpha\ n+1} & \text{if } f \le 0 \\ \mu\left|t_{Nn+1}\right| n^{trial}_{T\alpha\ n+1} & \text{if } f > 0 \end{cases} \tag{17}$$

with

$$n^{trial}_{T\alpha\ n+1} = \frac{t^{trial}_{T\alpha\ n+1}}{\left\|\mathbf{t}^{trial}_{Tn+1}\right\|} \tag{18}$$

For the both cases, the penalty method can be illustrated as a group of linear elastic springs that force the body back to the contact surface when overlapping or sliding occurs.

4. ALGORITHM FOR FRICTIONAL CONTACT

For solution a nonlinear equilibrium equation with inequality constraints (4) as a result of contact, we use a standard implicit method. In order to apply Newton's method for the solution system of equilibrium equation, a linearization of the contact contributions is necessary. In this paper, we do not state the linearization procedure for standard finite element formulation as well as the contact interface law for the normal and tangential part. It could be found in [7].

The tangent stiffness matrix for the normal contact is

$$\mathbf{K}_N = \varepsilon_N \mathbf{N}\mathbf{N}^T \tag{19}$$

The symmetric tangent stiffness matrix for stick condition is

$$\mathbf{K}^{stick}_T = \varepsilon_T \bar{a}_{\alpha\beta} \mathbf{D}^{\alpha} \mathbf{D}^{\beta T} \tag{20}$$

where

$$\mathbf{N} = \begin{Bmatrix} \bar{\mathbf{n}} \\ -H_1\bar{\mathbf{n}} \\ -H_2\bar{\mathbf{n}} \\ -H_3\bar{\mathbf{n}} \\ -H_4\bar{\mathbf{n}} \end{Bmatrix} \qquad \mathbf{T}_\beta = \begin{Bmatrix} \bar{\mathbf{a}}_\beta \\ -H_1\bar{\mathbf{a}}_\beta \\ -H_2\bar{\mathbf{a}}_\beta \\ -H_3\bar{\mathbf{a}}_\beta \\ -H_4\bar{\mathbf{a}}_\beta \end{Bmatrix} \qquad \mathbf{D}^{\alpha} = \bar{a}^{\alpha\beta}\mathbf{T}_\beta . \tag{21}$$

The linearization of $n^{trial}_{T\alpha\ n+1}$ gives (for details see [1])

$$\Delta\left(n^{trial}_{T\alpha\ n+1}\right) = \Delta\left(\frac{t^{trial}_{T\alpha\ n+1}}{\left\|\mathbf{t}^{trial}_{Tn+1}\right\|}\right) = \frac{1}{\left\|\mathbf{t}^{trial}_{Tn+1}\right\|}\left[\delta^{\beta}_{\alpha} - n^{trial}_{T\alpha\ n+1} n^{trial\ \beta}_{T\ n+1}\right]\Delta t^{trial}_{T\beta\ n+1} \tag{22}$$

The tangent stiffness matrix for slip condition is

$$\mathbf{K}_T^{slip} = \mu\varepsilon_N n^{trial}_{T\alpha\,n+1}\mathbf{D}^\alpha\mathbf{N}^T + \frac{\mu\varepsilon_N g_{Nn+1}}{\left\|\mathbf{t}^{trial}_{Tn+1}\right\|}\varepsilon_T\bar{a}_{\beta\gamma}\left[\delta^\beta_\alpha - n^{trial}_{T\alpha\,n+1}n^{trial\,\beta}_{T\,n+1}\right]\mathbf{D}^\alpha\mathbf{D}^{\gamma T} \tag{23}$$

The second term, the tangent matrix is non-symmetric, due to the Coulomb's friction can be viewed as a non-associative constitutive equation.

Frictional contact algorithm using penalty method is shown in Table 1.

Table 1. Frictional contact algorithm using the penalty method

LOOP over all contact segment k
(check for contact (6)) IF $g_N \le 0$ THEN
(the first iteration) IF $i=1$ THEN
set all active nodes to state stick,
$\mathbf{t}_{Tn+1}$ (18), compute matrix $\mathbf{K}_T^{stick}$
ELSE
Compute trial state: $t^{trial}_{T\alpha\,n+1}$ (19) and f^{trial}_{Tn+1} (20)
IF $f^{trial}_{Tn+1} \le 0$ THEN
$t_{T\alpha\,n+1} = t^{trial}_{T\alpha\,n+1}$, compute matrix $\mathbf{K}_T^{stick}$ (40)
GO TO (a)
ELSE
$t_{T\alpha\,n+1} = \mu\left|t_{Nn+1}\right| n^{trial}_{T\alpha\,n+1}$, compute matrix $\mathbf{K}_T^{slip}$ (43)
ENDIF
ENDIF
ENDIF
(a) END LOOP

4. EXAMPLE

For purposes of comparison, numerical example is taken from [6]. Hence this example can be used to verify whether the developed algorithm is able to represent stick/slip behavior correctly. An elastic block is pressed against a rough rigid foundation. Simultaneously to the vertical loading the block is pulled at right side by an uniformly normal stress (see Fig. 2). Material constants are: $E = 1000$ per lenght square, $\nu = 0.3$. The properties of the contact surface have been chosen as follows: $\varepsilon_N = 10^8$, $\varepsilon_T = 10^4$, friction coefficient $\mu = 0.5$. The block is discretized using 200 four-node isoparametric elements. It should be noted that using developed algorithm the total load can be applied in only one step.

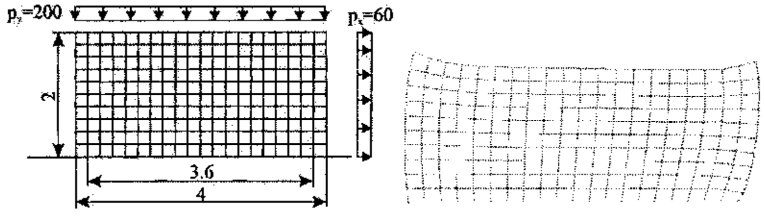

Fig. 2. Initial and deformed configuration

The computed normal contact pressure and tangential contact stress are shown in Fig. 3, and indicate good agreement between this solutions and the solutions shown in [6].

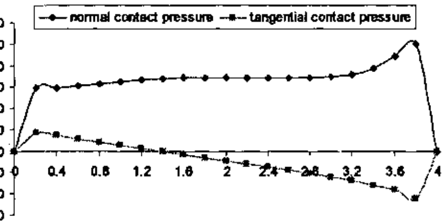

Fig. 3. Force – displacement relationship

3. CONCLUSIONS

A model for contact problem with friction, based on the penalty method, was presented. Due to the intrinsic similarity between friction and the classical elaso-plasticity [4], the constitutive model for friction can be constructed following the same formalism as in classical elaso-plasticity. The numerical example indicates a possibility of applying the developed method in the analysis of finite deformation problems.

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